Modelling Voltage-Demand Relationship on Power Distribution Grid for Distributed Demand Management

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***Abstract*— Most existing demand response or management algorithms require a dedicated communication infrastructure to coordinate actions of electricity users. However, the necessary communication infrastructures may not be available in many low-voltage (LV) networks around the world. On the other hand, implicit information on the state of the network is readily available at all times via measurements. In this paper we propose a stochastic modelling approach to estimate aggregate network demand from local voltage measurements at each household using a gamma distribution. The model suggests a linear relationship between the expected value of network demand and voltages at households in the network. We propose a set of illustrative distributed demand control algorithms that allow making decisions based on local information only. Depending on the nature of different appliances, the algorithms either shift the entire demand block to another time (for deferable loads such as driers) or alter the consumption rate of an appliance continuously (for granular loads such as electric vehicles). We illustrate via simulations that the stochastic model captures the actual relationship between voltage and demand. The resulting demand management algorithms are efficient in reducing demand peaks without reducing the overall consumption. Moreover, the lack of explicit communication requirements makes the algorithms scalable and readily applicable to most LV networks.**

I. INTRODUCTION

Electricity consumption has a time varying nature. Typically, the difference between peak and off-peak electricity consumption is significant. Such differences are currently compensated through control of generators. As a consequence, during peak hours, generators are able to charge scarcity prices that are significantly higher than the average price which also drives up the retail price. In Victoria, Australia, 2014, the highest wholesale price which occurred on one of the summer days was 70 times more than the average price and the highest 5-minute dispatch price is as high as 300 times the average [1]. In addition, the expensive grid infrastructure, which is designed to withstand peak demand, is heavily under-utilized for off-peak hours. In Victoria, Australia, 2012, more than 25% of the grid is only used for less than 2% of the time in the year [2]. That is billion-dollar worth of infrastructure sitting idle for almost all the time, and it is getting worse over time [2].

Demand management or demand response is proposed as an efficient way to control electricity demand, flatten overall consumption, reduce whole sale electricity price volatility and improve grid asset utilization. Some implemented algorithms such as direct load control (DLC) and time-of-use tariff (TOU) have obvious drawbacks. DLC cuts off user consumption in need without accounting for the comfort of consumers and TOU does not reflect the real demand dynamics of the grid [3]. Recently, some algorithms using dynamic pricing (or a price-alike signal) or centralized control have been proposed to manage energy consumption at the demand side [4]–[9]. One feature the algorithms share in common is that a dedicated single directional or bi-directional communication channel needs to be present for the algorithms to work. However, such infrastructure does not commonly exist which makes the algorithms not readily applicable. More recently, some control algorithms that make decisions based only on local measurements have been proposed [10], [11]. However, the conclusions are rather heuristic and the underlying principles of using household voltages as network demand indicators are not analysed in detail. In this paper, we model the stochastic nature of LV networks, in particular the last mile, and then develop demand management algorithms based on it.

Stochastic analysis is a popular tool in demand forecasting, demand modelling for the purpose of LV network design [12]–[14]. A beta distribution model is proposed in [12] to estimate household load profiles such that the peak demand can be estimated and transformers can be sized accordingly. Such approach has been adopted in the electricity network design in South Africa [12]. Later, a gamma distribution model is shown to be more accurate in terms of capturing household half-hourly demand patterns via Monte Carlo simulations [13]. The authors have also shown that parameters of probability density functions can be extracted from known data-set and the model is calibrated against varying temperature. Such stochastic modelling can be implemented on a finer level where load profiles of different appliances within a household are modelled as different random variables [14]. Instead of a single random variable, the demand profile of a household is now a combination of several random variables. The results have also been shown to well match the actual demand profiles. In this paper, we use a simple gamma distribution to model household demand patterns and the parameters for the distribution are extracted from actual meter data. With this probability distribution of demand profiles, using classic circuit theory, we correlated demand and voltages in a distribution network such that the probability distribution of household voltages under a given network demand can also be derived. Using that information,

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the power-line itself could then act as a communication channel and household voltage measurements can be used as a signal for demand management coordination.

The main contributions of this paper are the following:

- We analyse the stochastic nature of electricity demand of households at any time instance and propose a gamma distribution based model to correlate network demand and household voltages.
- Using the stochastic model, we develop two distributed control algorithms for demand management based on household voltage measurements. The algorithms are tailored to suit the physical energy consumption requirements of different appliance categories.
- The model and algorithms are verified via detailed modelling of a real Australian suburban network under normal operating condition.

The rest of the paper is organized as follows. Section II introduces the network modelling as well as the stochastic approach for network demand estimation using household voltages. Section III proposes two illustrative algorithms for demand management based on the stochastic model. Section IV verifies the performances of the proposed models and algorithms based on simulations. The work is concluded in Section V and some possible future work is proposed.

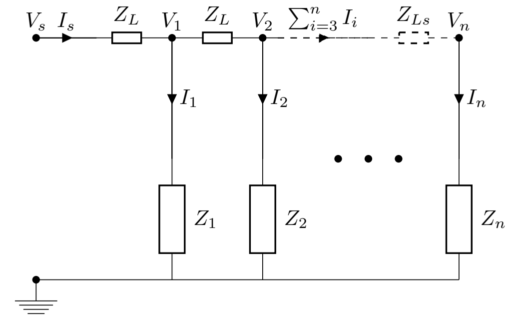

Fig. 1. System diagram for a single branch network

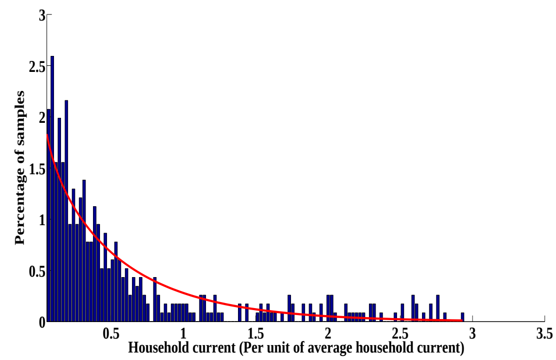

Fig. 2. Distribution of households demand for a distribution network at a time instance fitted by a gamma distribution

II. Voltage-demand relationship modelling

A. Single branch network

The focus of the paper is on the last mile of three-phase radial networks where each house is connected to a single phase. Almost all Australian networks are of such configuration. Different from a transmission network, in a typical last mile residential distribution network, loads are predominately active power loads and therefore the power factors are usually very high. Hence, we start the analysis by assuming a balanced network with unity power factor. Such assumptions will be violated in Section IV to test the tolerance of proposed model. We start by looking at networks having only one branch and then extend the observation to branched networks. Figure 1 shows the schematic of one of three phases. There are in total n houses connected on this phase. Households are modelled as current loads where $I_x,\ x = 1, 2, ..., n$ depicts the demand of house x in terms of current. The total demand in the network is also defined in terms of current, I_s, and the voltage at the source side, V_s, is assumed to be constant. We proceed by making the assumption that the line impedances between any two adjacent houses are the same denoted by Z_L, which will be true for many residential networks where all properties have roughly the same sizes. For unbalanced networks, the formulation will be slightly different but the fundamentals remain. Using circuit theory, the relationship between voltages and demand is expressed as (1).

$$
\begin{aligned}
V_1 &= V_s - Z_L \sum_{i=1}^{n} I_i \\
V_2 &= V1 - Z_L \sum_{i=2}^{n} I_i = V_s - Z_L \sum_{i=1}^{n} I_i - Z_L \sum_{i=2}^{n} I_i \\
V_3 &= V_s - Z_L(\sum_{i=1}^{n} I_i + \sum_{i=2}^{n} I_i + \sum_{i=3}^{n} I_i) \\
&\vdots \\
V_n &= V_s - Z_L(\sum_{i=1}^{n} I_i + \sum_{i=2}^{n} I_i + \sum_{i=3}^{n} I_i + \cdots + I_n)
\end{aligned}
\tag{1}
$$

As being implicitly assumed in [11], if the demand of each household is identical to the others at any time. i.e. $I_x = I_s/n,\ x = 1, 2, ..., n$. Equation set (1) can be quickly reduced. The total demand in the network and local voltage measurements are correlated via the following equations for household $x,\ x = 1, 2, ..., n$.

$$
\begin{aligned}
V_x &= V_s - Z_L(n\frac{I_s}{n} + (n-1)\frac{I_s}{n} + \cdots + (n-x+1)\frac{I_s}{n}) \\
&= V_s - Z_L \frac{I_s}{n} \frac{(2n-x+1)x}{2}
\end{aligned}
\tag{2}
$$

From (2), for any household x in the network, the relationship between the local voltage V_x and total current in the network I_s is linear. However, the assumption does not hold as the demands of individual households will rarely be identical. To understand what the demand is like, we have collected and plotted the real household demand in terms of current (normalized with respect to the average demand) at any time instance in the year 2013 for a network located in Melbourne, Australia. The histogram of household currents at a random time instance is shown in Figure 2. The distribution has a tall head and a long tail which can be fitted by a gamma probability density function (shown as a continuous curve in Figure 2). Therefore, we assume that the demand of houses be mutually independent variables $(A_x, x = 1, 2, ..., n)$ which satisfy a gamma distribution that is described as follows:

$$A_x \sim \Gamma(k, \theta) \tag{3}$$

$$E[A_x] = k\theta := \frac{I_s}{n} \tag{4}$$

$$Var[A_x] = k\theta^2 := \theta\frac{I_s}{n} \tag{5}$$

where the shape parameter k and the scale parameter θ describes the distribution which can be obtained by fitting the histogram of actual demand data. The mean of the distribution is $E[A_x]$ and the variance is $Var[A_x]$. Note that the distribution is time varying and I_s is also time varying. The notations in this paper capture the behaviour of the network at a time instance. We found that at a different time in a day or in a year, the value of k changes according to the total demand I_s but the value of θ could stay unchanged. In other word, gamma functions can be fitted such that for any time instance, θ is a constant. Now let U_x be the random voltage at house x at a time instance, (1) can be rewritten as follows for user x, $x = 1, 2, ..., n$:

$$U_x = V_s - Z_L B_x \tag{6}$$

$$\begin{aligned} B_x &= x(A_x + A_{x+1} + \cdots + A_n) \\ &\quad + (x-1)A_{x-1} + \cdots + 2A_2 + A_1 \end{aligned} \tag{7}$$

Note that the distribution of random variable B_x, which is a linear combination of independent gamma variables, is still an open research question. However, a widely used method, the Welch-Satterthwaite method [15], approximates B_x using a gamma random variable $\hat{B}_x \sim \Gamma(k_{\hat{B}_x}, \theta_{\hat{B}_x})$ for $x = 1, 2, ..., n$ that can be written as follows:

$$E[\hat{B}_x] = k\theta\frac{(2n - x + 1)x}{2} \tag{8}$$

$$Var[\hat{B}_x] = k\theta^2\frac{-4x^3 + 6nx^2 + 3x^2 + x}{6} \tag{9}$$

$$k_{\hat{B}_x} = E^2[\hat{B}_x]/Var[\hat{B}_x] \tag{10}$$

$$\theta_{\hat{B}_x} = Var[\hat{B}_x]/[\hat{B}_x] \tag{11}$$

B. Multiple-branch network

We now extend the findings to branched networks with multiple feeders. Figure 3 shows one phase of a network with m branches. The problem can be solved in two stages. Firstly, assuming that each branch is a single load, the problem will be transformed into a single branch problem as in the previous subsection where loads have different

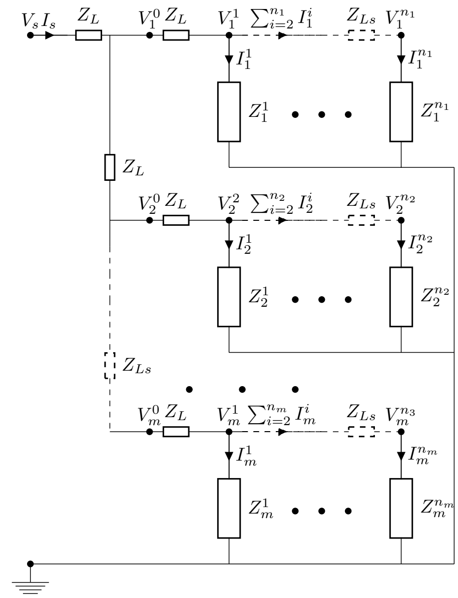

Fig. 3. System diagram for a multiple branch network

characteristics. Assume that the number of houses on branch $1, 2, ..., m$ is $n_1, n_2, ..., n_m$, using (3) for the household demand, the aggregated demand on each branch is a gamma random variable $M_y \sim \Gamma(k_y, \theta), y = 1, 2, ..., m$ which can be described as follows:

$$k_y = n_y k \tag{12}$$

$$E[M_y] = n_y k\theta \tag{13}$$

$$Var[M_y] = n_y k\theta^2 \tag{14}$$

Now let U_y^0 be the random voltage at the first node of branch $y, y = 1, 2, ..., m$. Apply (6) and (7) here, we have:

$$\begin{aligned} U_y^0 &= V_s - Z_L B_y^0 \\ B_y^0 &= y(M_y + M_{y+1} + \cdots + M_m) \\ &\quad + (y-1)M_{y-1} + \cdots + 2M_2 + M_1 \end{aligned} \tag{15}$$

Again, B_y^0 is a linear combination of random gamma variables which can be approximated using a gamma distributed random variable $\hat{B}_y^0$. The parameters of $\hat{B}_y^0 \sim \Gamma(k_{\hat{B}_y^0}, \theta_{\hat{B}_y^0})$ for $y = 1, 2, ..., m$ are given as follows:

$$\begin{aligned} E[\hat{B}_y^0] &= k\theta(y(n_y + n_{y+1} + \cdots + n_m) \\ &\quad + (y-1)n_{y-1} + \cdots + 2n_2 + n_1) \\ Var[\hat{B}_y^0] &= k\theta^2(y^2(n_y + n_{y+1} + \cdots + n_m) \\ &\quad + (y-1)^2 n_{y-1} + \cdots + 4n_2 + n_1) \\ k_{\hat{B}_y^0} &= E^2[\hat{B}_y^0]/Var[\hat{B}_y^0] \\ \theta_{\hat{B}_y^0} &= Var[\hat{B}_y^0]/E[\hat{B}_y^0] \end{aligned}$$

Secondly, if the voltage at the first node of each branch (V_1^0, V_2^0,...,V_m^0) is constant, the problem on each branch will be identical to the single branch scenario. However, since the voltage at the first node of each branch is now a random variable itself (U_1^0, U_2^0,...,U_m^0), the problem will be changed slightly. We can write the following for house x on branch y:

$$U_y^x = U_y^0 - Z_L B_y^x = V_s - Z_L B_y^0 - Z_L B_y^x \quad (16)$$

B_y^x can be approximated by a gamma random variable $\hat{B}_y^x$ described as follows:

$$E[\hat{B}_y^x] = k\theta \frac{(2n_y - x + 1)x}{2}$$
$$Var[\hat{B}_y^x] = k\theta^2 \frac{-4x^3 + 6n_y x^2 + 3x^2 + x}{6}$$

Define the voltage drop at house x on branch y as $\Delta U_y^x = V_s - U_y^x = Z_L B_y^0 + Z_L B_y^x$. It can be approximated as $\Delta\hat{U}_y^x = Z_L \hat{B}_y^0 + Z_L \hat{B}_y^x$, then $\Delta\hat{U}_y^x \sim \Gamma(k_{\Delta\hat{U}_y^x}, \theta_{\Delta\hat{U}_y^x})$ can be described by the following parameters:

$$\begin{aligned} E[\Delta\hat{U}_y^x] &= kZ_L\theta(\frac{(2n_y - x + 1)x}{2} + y(n_y + \cdots + n_m) \\ &+ (y-1)n_{y-1} + \cdots + 2n_2 + n_1) \quad (17) \\ Var[\Delta\hat{U}_y^x] &= kZ_L^2\theta^2(\frac{-4x^3 + 6n_y x^2 + 3x^2 + x}{6} \\ &+ y^2(n_y + n_{y+1} + \cdots + n_m) \\ &+ (y-1)^2 n_{y-1} + \cdots + 4n_2 + n_1) \quad (18) \\ k_{\Delta\hat{U}_y^x} &= E^2[\Delta\hat{U}_y^x]/Var[\Delta\hat{U}_y^x] \\ \theta_{\Delta\hat{U}_y^x} &= Var[\Delta\hat{U}_y^x]/[\Delta\hat{U}_y^x] \end{aligned}$$

Then, using (4) and (5), we can simplify (17) and (18) as:

$$E[\Delta\hat{U}_y^x] = K_y^x Z_L I_s \quad (19)$$
$$Var[\Delta\hat{U}_y^x] = \theta W_y^x Z_L^2 I_s \quad (20)$$

where K_y^x and W_y^x are house specific constants depending purely on the topology of network and household location. K_y^x and W_y^x can be written as follows:

$$\begin{aligned} K_y^x &= \frac{1}{n}(\frac{(2n_y - x + 1)x}{2} + y(n_y + n_2 + \cdots + n_m) \\ &+ (y-1)n_{y-1} + \cdots + 2n_2 + n_1) \quad (21) \\ W_y^x &= \frac{1}{n}(\frac{-4x^3 + 6n_y x^2 + 3x^2 + x}{6} \\ &+ y^2(n_y + n_{y+1} + \cdots + n_m) \quad (22) \\ &+ (y-1)^2 n_{y-1} + \cdots + 4n_2 + n_1) \end{aligned}$$

Therefore, we have obtained a linear expression as in (19) to relate the expected values of individual household voltages and total network demand. Note that the parameters of the linear expressions are specific to individual households. Therefore, no houses are intentionally advantaged or disadvantaged under such a paradigm. The variance of such an approximation is also linear to the total demand as shown in (20) . Most distributed demand response algorithms require the total network demand to be broadcast either in its direct form or in the form of a dynamic price via a dedicated communication channel. The above analysis shows that the power line itself can be such communication channel: household voltages can be used as the dynamic price signals to coordinate demand management of various users. To further justify the model under actual operating conditions, we collect and plot the network total current versus individual households' voltages for a real Victorian suburban network. Figure 4 shows the plot of a randomly selected household over a 24 hour period sampled every 15 minutes. Each dot on the plot indicates the local voltage at that house and the corresponding network total current at a sample time. The scatter plot is well fitted with a straight line and the approximation errors increases as total current increases, which justifies our theoretic model. Such errors can then be compensated through well designed controllers using feedback.

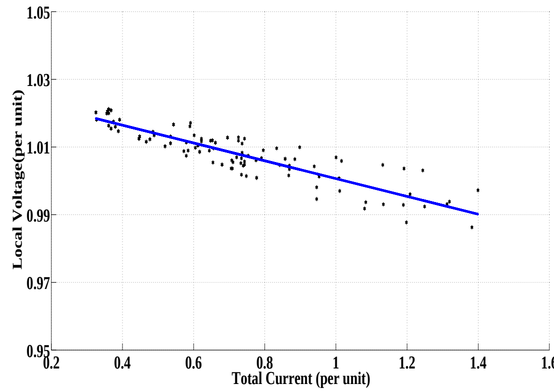

Fig. 4. Relationship between network total current and local voltage of a house in the network

III. Distributed demand management algorithm examples

The main goal of demand management algorithms is to move the peak demand to off-peak periods such that the overall power consumption pattern is flattened. By doing so, the grid infrastructure is better utilised and wholesale electricity price volatility is avoided. In addition, a flat consumption pattern leads to a more stable voltage profile which improves power quality in a distribution network.

In order to illustrate our voltage-demand model, we propose two illustrative example algorithms for demand management without any explicit communication requirements. If the network topology and household locations are known to each house, a linear controller can be developed for demand management. However, usually the topology is not known to individuals. Therefore, we propose a linear-increase-linear-decrease style controller for each household to iteratively reach its demand management goal.

Depending on load types, the controller will be slightly different. We first of all categorize the loads into two categories:

1) Storable demand loads: The first category of loads includes those appliances that are relatively robust to variations in power input. Some typical examples include air-conditioning (AC) units, space heaters, electrical water heaters and electric vehicles (EVs). For AC units and fridges, the objective is to keep interior temperature within a desired range. Therefore, an interruption for several minutes followed by a higher cooling output or vice versa can still sustain desired temperature and provide scheduling flexibility. For EVs and water heaters, the flexibility is even higher. Such appliances can be interrupted for hours without affecting the quality of service.

2) Shiftable demand loads: The second category of loads includes those whose demand can be shifted but not easily interrupted. Examples of such loads include washing machines, driers and dish washers. These appliances can be delayed until there is increased capacity in the network before they are turned on. However, once these appliances are turned on, they should not be interrupted until their duty cycles finish.

Algorithms 1 and 2 are proposed for storable loads and shiftable loads respectively. Algorithm 1 is a rate-based controller which controls the current flowing into an appliance. Current increases or decreases linearly according to the voltage measurements. Algorithm 2 is a probability-based controller that controls the probability for an appliance to be switched on or off. Once switched on, the appliance will complete its full duty cycle without interruption.

Algorithm 1 Storable loads management

```
Input: V(t), V_min, V_max, τ
Output: I(t)
1: I(0) ← 0, t ← 1                                  ▷ initialization
2: I(t) ← I(t−1) + I_rated * (V(t) − V_min)/(V_max − V_min)
3: I(t) ← min{I(t), I_rated}
4: I(t) ← max{I(t), 0}
5: wait for (τ), t ← t + 1
6: goto (step 2)
```

Algorithm 2 Shiftable loads management

```
Input: V(t), V_min, V_max, τ
Output: I(t)
1: I(0) ← 0, t ← 1                                  ▷ initialization
2: p(t) ← p(t − 1) + (V(t) − V_min)/(V_max − V_min)
3: p(t) ← min{p(t), 1}
4: p(t) ← max{p(t), 0}
5: if rand(0,1)<p(t) then
6:     I(t) = I_rated
7:     run full duty cycle
8:     end
9: else
10:    I(t) = 0
11:    wait for (τ), t ← t + 1
12:    goto (step 2)
13: end if
```

In the algorithms, I_{rated} represents the current rating of the appliance, $V(t)$ denotes household voltage at time t, $I(t)$ denotes the current consumption at time t and $p(t)$ (if applicable) denotes the probability of having $I(t)$; V_{min}, V_{max} are minimum and maximum threshold voltages of a household which can be obtained from historical measurements. The interval length τ between iterations should be carefully chosen for the algorithm to obtain desired outcome. Statistically, the two control algorithms will have the same control result. Let the demand expected value or expectation of a household at time t be

$$E_t[I] = E_{t-1}[I] + \sum_{i=1}^{\infty} \Delta I_i(t) * p_i(t) \tag{23}$$

where $\Delta I_i(t)$ is a possible increment in current that an appliance could take and $p_i(t)$ is the probability for $\Delta I_i(t)$ to be taken. Assume that one appliance of each kind is about to make a demand decision at time t under exactly the same grid condition.

For storable loads, using Algorithm 1, we have the following

$$p_i(t) = \begin{cases} 1, & \text{if } \Delta I_i(t) = I_{max} * \frac{V(t)-V_{min}}{V_{max}-V_{min}} \\ 0, & \text{otherwise} \end{cases}$$

For storable loads, using Algorithm 2, we have the following

$$p_i(t) = \begin{cases} \frac{V(t)-V_{min}}{V_{max}-V_{min}}, & \text{if } \Delta I_i(t) = I_{max} \\ 0, & \text{otherwise} \end{cases}$$

For the two cases, using (23), the expected values of demand for time t are identical as shown in the following equation. In other words, even though the two controllers act differently, they would essentially yield the same outcome statistically.

$$E_t[I] = E_{t-1}[I] + I_{max} * \frac{V(t) - V_{min}}{V_{max} - V_{min}} \tag{24}$$

IV. Simulations

To illustrate the performance of the illustrative algorithms under real operating conditions, we use a model of a Victoria suburban three-phase residential distribution network in Simulink based on the actual topology and specifications of the real network. Then we simulate the algorithms using real demand data measured in this network. The network has 114 households supplied by a $200kVA$ transformer. We assume that 50% of the load in the network is inflexible load which cannot be managed, 25% of the loads are shiftable and 25% are storable. Figure 5 shows the total demand before and after management. Without demand management. The yellow area represents inflexible loads. The area between the yellow area and the two lines shows the flexible load patterns before and after demand management. Without management, there is a peak which is several times higher than the valley (peak to average ratio is 1.89). Using our demand management algorithm, the peak demand is successfully shifted to other periods and the overall demand pattern becomes much flatter (peak to average ratio is 1.11). Figure 6 shows the voltage profile of a random household in the network. Voltage variations are also well reduced with the algorithms.

V. Conclusion and Future Work

In this paper, we propose a stochastic model to describe the correlation between voltages and demand in LV networks. Based on such a model, we propose two distributed demand

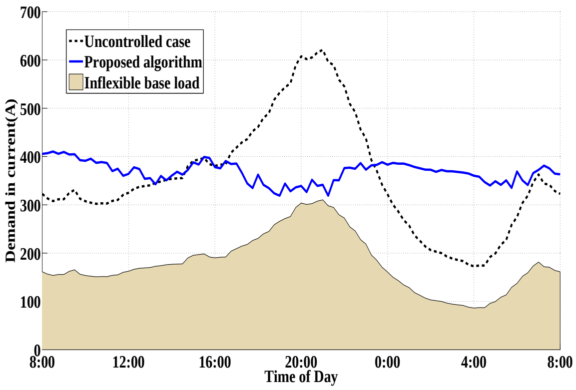

Fig. 5. Demand profile of the network with and without management.

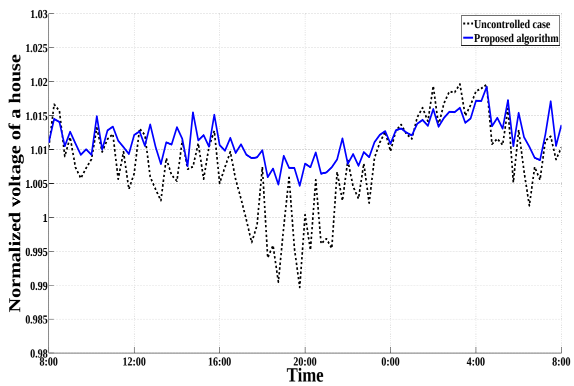

Fig. 6. Voltage profile of a house with and without management.

management algorithms targeted at shiftable and storable appliances. Via simulation on a real Australian suburban network, using real demand profiles, the model is shown to capture actual voltage-demand relationship and the algorithms are shown to be effective for peak shifting. Most importantly, the algorithms use only local information, voltage, for decision making and do not require any additional communication infrastructure. Future work includes building stochastic and/or linear controllers for more accurate control, application to networks with high solar panel penetration, application to non-radial networks and application to heavily unbalanced networks, detailed study on fairness, taken service line impedance into account and modelling loads as impedances instead of currents.

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